Marriage Institutions and Sibling Competition: Online Theory Appendix

The One-Daughter Problem

Let $V_1(a)$ be the expected value of the daughter at age a. Let $v_1^A(q,a)$ be the expected value after a representative of a groom of quality q has arrived, but before any offer takes place. And let $v_1^N(a)$ be the expected value in the event that a representative does not arrive. Then:

$$\begin{aligned} V_1(a) &= \lambda \mathbb{E} v_1^A(q,a) + (1-\lambda)\, v_1^N(a) \\ v_1^A(q,a) &= \max\{\pi(a)q + (1-\pi(a))V_1(a+1), V_1(a+1)\} \\ v_1^N(a) &= V_1(a+1) \end{aligned}$$

Because the problem is non-stationary, the optimal policy involves setting an age-dependent reservation quality $q_1(a)$. We solve for $q_1(a)$ using backward induction. At $a = \bar{a}$, $V_1(a+1) = 0$ because $\pi(a+1) = 0$. Thus, $q_1(\bar{a}) = 0$. At $a \in [\underline{a}, \overline{a})$:

$$\mathbb{E} v_1^A(a) = \int_{\underline{Q}}^{q_1(a)} q_1(a) dF(\tilde{q}) + \int_{q_1(a)}^{\overline{Q}} \{\pi(a)\tilde{q} + (1-\pi(a))\, q_1(a)\}\, dF(\tilde{q})$$

and:

$$q_1(a) = \lambda \mathbb{E} v_1^A(q, a+1) + (1-\lambda)\, v_1^N(a+1)$$

which yields the recursive solution:

$$q_1(a) - q_1(a+1) = \lambda \pi(a+1)\,(1 - F[q_1(a+1)])\,(\mathbb{E}[q|q > q_1(a+1)] - q_1(a+1))$$

Let $\psi(\tilde{q}) = (1 - F[\tilde{q}])\,(\mathbb{E}[q|q > \tilde{q}] - \tilde{q})$, so that $q_1(a) - q_1(a+1) = \lambda \pi(a+1)\psi\,(q_1(a+1))$. This transition function is positive, so $q_1(a) > q_1(a+1)$ for all $a \in [\underline{a}, \overline{a})$. Also note that:

$$\frac{d\psi}{d\tilde{q}} = -f\,(\tilde{q})\,(\mathbb{E}[q|q > \tilde{q}] - \tilde{q}) + (1 - F[\tilde{q}]) \left(\frac{d\mathbb{E}[q|q > \tilde{q}]}{d\tilde{q}} - 1 \right)$$

assuming that the density of F exists. The first term is strictly negative, and the second term is weakly negative due to log concavity, so $\psi(\cdot)$ decreases in $\tilde{q}$.[1]

[1] If q is log-concavely distributed, then $\frac{d\mathbb{E}[q|q>\tilde{q}]}{d\tilde{q}} \leq 1$ (Goldberger 1983).

The Two-Daughter Problem with a Marriage-by-Birth-Order Rule

Let a be the elder daughter's age, and let Δ be the age gap between daughters. The younger daughter is prohibited from marrying before her elder sister marries. Let $V_2(a,\Delta)$ be the expected value of the two unmarried daughters. Let $v_2^A(q,a,\Delta)$ be their expected value after a representative of a groom of quality q has arrived, but before any offers take place. And let $v_2^N(a,\Delta)$ be their expected value in the event that a representative does not arrive. Then for $a \in [\underline{a}+\Delta, \overline{a}]$:

$$\begin{aligned} V_2(a,\Delta) &= \lambda \mathbb{E} v_2^A(q,a,\Delta) + (1-\lambda)\, v_2^N(a,\Delta) \\ v_2^A(q,a,\Delta) &= \max\left\{\pi(a)\left(q + V_1(a-\Delta+1)\right) + (1-\pi(a))V_2(a+1,\Delta), V_2(a+1,\Delta)\right\} \\ v_2^N(a,\Delta) &= V_2(a+1,\Delta) \end{aligned}$$

Let $q_2(a,\Delta)$ be the reservation value for the elder daughter's groom. At $a=\bar{a}$, if the family fails to marry the elder daughter, neither daughter can ever be married in the future. Thus, $v_2^A(q,\overline{a},\Delta) = \max\{\pi(\overline{a})\left(\mathbb{E}\left[q|q \geq q_2(a,\Delta)\right] + q_1(\bar{a}-\Delta)\right), 0\}$. As a result, $q_2(\bar{a},\Delta)$ solves $\mathbb{E}\left[q|q \geq q_2(\bar{a},\Delta)\right] = -q_1(\bar{a}-\Delta)$, so that $q_2(\bar{a},\Delta) < 0$. For $a \in [\underline{a}+\Delta, \overline{a})$:

$$\mathbb{E} v_2^A(q,a,\Delta) = \int_{\underline{Q}}^{q_2(a,\Delta)} q_2(a,\Delta) dF(\tilde{q}) + \int_{q_2(a,\Delta)}^{\overline{Q}} \left\{\pi(a)\tilde{q} + (1-\pi(a) q_2(a,\Delta)\right\} dF(\tilde{q}) + q_1(a-\Delta)$$

and:

$$\begin{aligned} q_2(a,\Delta) &= \lambda \mathbb{E} v_2^A(q,a+1,\Delta) + (1-\lambda)\, v_2^N(a+1,\Delta) - q_1(a-\Delta) \\ &= \lambda \mathbb{E} v_2^A(q,a+1,\Delta) + (1-\lambda)\, V_2(a+2,\Delta) - q_1(a-\Delta) \\ &= \lambda \mathbb{E} v_2^A(q,a+1,\Delta) + (1-\lambda)\left(q_2(a+1,\Delta) + q_1(a-\Delta+1)\right) - q_1(a-\Delta) \end{aligned}$$

which yields the following transition function:

$$q_2(a,\Delta) - q_2(a+1,\Delta) = \lambda\pi(a+1)\psi\left(q_2(a+1,\Delta)\right) - \lambda\pi(a-\Delta+1)\psi\left(q_1(a-\Delta+1)\right)$$

This transition function equals the transition function of the one-daughter case minus $\lambda\pi(a-\Delta+1)\psi\left(q_1(a-\Delta+1)\right)$. So $q_2(a,\Delta) < q_1(a)$ for $a \in [\underline{a}+\Delta, \overline{a}]$. For $a \in [\underline{a}, \underline{a}+\Delta)$, the transition function reverts to the one-daughter case, but since $q_2(\underline{a}+\Delta,\Delta) < q_1(\underline{a}+\Delta)$, $q_2(a,\Delta) < q_1(a)$ holds for all a.

Proof of Result

(a) A girl with a younger sister has higher cumulative marriage risk at any given age.

Let $P_o(a)$ be the cumulative probability of marriage by age a for an only daughter:

$$P_o(a) = \sum_{\tilde{a}=\underline{a}}^{a} \lambda\pi(\tilde{a}) \left(1 - F\left[q_1(\tilde{a})\right]\right) \left(1 - P_o(\tilde{a} - 1)\right)$$

Let $P_e(a, \Delta)$ be the cumulative probability of marriage by age a for an elder daughter with a younger sister aged $a - \Delta$:

$$P_e(a, \Delta) = \sum_{\tilde{a}=\underline{a}}^{a} \lambda\pi(\tilde{a}) \left(1 - F\left[q_2(\tilde{a}, \Delta)\right]\right) \left(1 - P_2(\tilde{a} - 1, \Delta)\right)$$

Because $q_2(a, \Delta) < q_1(a)$, the failure rate for an elder daughter is always higher than that for an only daughter: $1 - \lambda\pi(a)\left(1 - F\left[q_e(a, \Delta)\right]\right) > 1 - \lambda\pi(a)\left(1 - F\left[q_1(a)\right]\right)$. As a result $P_e(a, \Delta) > P_o(a)$ for all $a > \underline{a}$ and for all Δ. ■

(b) A girl with an elder sister has lower cumulative marriage risk at any given age.

Let $P_y(a, \Delta)$ be the cumulative probability of marriage by age a for a younger daughter with an elder sister aged $a + \Delta$:

$$P_y(a, \Delta) = \sum_{\tilde{a}=\underline{a}}^{a} \lambda\pi(\tilde{a}) \left(1 - F\left[q_1(\tilde{a})\right]\right) \left(1 - P_y(\tilde{a} - 1, \Delta)\right) P_e(\tilde{a} + \Delta - 1, \Delta)$$

This expression is identical to $P_o(a)$, except that failure rates are multiplied by the probability that the elder sister is unmarried when the younger sister is aged a, $1 - P_e(\tilde{a} + \Delta - 1, \Delta)$. This quantity is less than 1, so $P_o(a) > P_y(a, \Delta)$ for all $a > \underline{a}$ and for all Δ. ■

(c) A girl with either an elder or younger sister has lower expected spousal quality.

Let $\hat{q}_o$ denote expected spousal quality for an only daughter, and let $\hat{q}_e(\Delta)$ and $\hat{q}_y(\Delta)$ denote expected spousal quality for the elder and younger daughters from a two-daughter family with age gap Δ. Assume that spinsterhood is equivalent to a spouse of quality zero. For an only daughter, we have:

$$\hat{q}_o = \sum_{\tilde{a}=\underline{a}}^{\bar{a}} \lambda\pi(\tilde{a}) \left(1 - F\left[q_1(\tilde{a})\right]\right) \left(1 - P_o(\tilde{a}-1)\right) \mathbb{E}\left[q|q > q_1(\tilde{a})\right]$$

For a younger daughter, we have:

$$\hat{q}_y(\Delta) = \sum_{\tilde{a}=\underline{a}}^{\bar{a}} \lambda\pi(\tilde{a}) \left(1 - F\left[q_1(\tilde{a})\right]\right) \left(1 - P_y(\tilde{a}-1,\Delta)\right) P_e(\tilde{a}+\Delta-1,\Delta)\mathbb{E}\left[q|q > q_1(\tilde{a})\right]$$

Because $P_o(a) > \left(1 - P_y(a-1,\Delta)\right) P_e(a+\Delta-1,\Delta)$ for all $a > \underline{a}$ and for all Δ, $\hat{q}_y(\Delta) > \hat{q}_o$ for all Δ.

To prove that an elder daughter has lower expected quality than an only daughter, note that:

$$\begin{aligned}
\hat{q}_o &= \Pr\left[\text{marry at age } \underline{a}\right] \mathbb{E}\left[q|\text{marry at age } \underline{a}\right] + \left(1 - \Pr\left[\text{marry at age } \underline{a}\right]\right) \mathbb{E}\left[q|\text{not marry at age } \underline{a}\right] \\
&= \lambda\pi(\underline{a}) \left(1 - F\left[q_1(\underline{a})\right]\right) \mathbb{E}\left[q|q > q_1(\underline{a})\right] + \left\{1 - \lambda\pi(\underline{a}) \left(1 - F\left[q_1(\underline{a})\right]\right)\right\} \mathbb{E}\left[q|\text{not marry at age } \underline{a}\right] \\
&= V_1(\underline{a}) = \lambda\pi(\underline{a})\psi\left(q_1(\underline{a})\right) + q_1(\underline{a})
\end{aligned}$$

and by similar logic:

$$\hat{q}_e(\Delta) = V_2(\underline{a},\Delta) - V_1(\underline{a}-\Delta) = \lambda\pi(\underline{a})\psi\left(q_2(\underline{a},\Delta)\right) + q_2(\underline{a},\Delta)$$

Because $q_2(\underline{a}-1,\Delta) < q_1(\underline{a}-1)$, we have that $\hat{q}_e(\Delta) < \hat{q}_o$ for all Δ.■

Extension: The Two-Daughter Problem with no Marriage-by-Birth-Order Rule

In this setup, the family faces no institutional constraint. It receives the groom's representative and then decides which daughter to offer, if any. For $a \in [\underline{a}+\Delta, \bar{a}]$, when both daughters are marriageable:

$$\begin{aligned}
V_2(a,\Delta) &= \lambda\mathbb{E}v_2^A(q,a,\Delta) + (1-\lambda)\, v_2^N(a,\Delta) \\
v_2^A(q,a,\Delta) &= \max\left\{\pi(a)\left(q + V_1(a-\Delta+1)\right) + (1-\pi(a))V_2(a+1,\Delta),\right. \\
&\qquad \left.\pi(a-\Delta)\left(q + V_1(a+1)\right) + (1-\pi(a-\Delta))V_2(a+1,\Delta), V_2(a+1,\Delta)\right\} \\
v_2^N(a,\Delta) &= V_2(a+1,\Delta)
\end{aligned}$$

The three terms in $v_2^A(q,a,\Delta)$ represent the values of offering the elder daughter, offering the younger daughter, and offering neither, respectively. The family prefers offering the elder daughter to offering neither if:

$$\begin{aligned}\pi(a)\left(q+V_1(a-\Delta+1)\right)+(1-\pi(a))V_2(a+1,\Delta) &\geq V_2(a+1,\Delta)\\ q &\geq q_2^{en}(a,\Delta)\equiv V_2(a+1,\Delta)-V_1(a-\Delta+1)\end{aligned}$$

It prefers offering the younger daughter to offering neither if:

$$\begin{aligned}\pi(a-\Delta)\left(q+V_1(a+1)\right)+(1-\pi(a-\Delta))V_2(a+1,\Delta) &\geq V_2(a+1,\Delta)\\ q &\geq q_2^{yn}(a,\Delta)\equiv V_2(a+1,\Delta)-V_1(a+1)\end{aligned}$$

It prefers offering the younger daughter to offering the elder if:

$$\begin{gathered}\pi(a-\Delta)\left(q+V_1(a+1)\right)+(1-\pi(a-\Delta))V_2(a+1,\Delta)\\ \geq \pi(a)\left(q+V_1(a-\Delta+1)\right)+(1-\pi(a))V_2(a+1,\Delta)\\ q\geq q_2^{ye}(a,\Delta)\equiv \tfrac{\pi(a)V_1(a-\Delta+1)-\pi(a-\Delta)V_1(a+1)}{(\pi(a-\Delta)-\pi(a))}+V_2(a+1,\Delta)\end{gathered}$$

Recall that $q_1(a)=V_1(a+1)$. Then $q_2^{en}(a,\Delta)=V_2(a+1,\Delta)-q_1(a-\Delta)$, implying that $V_2(a+1,\Delta)=q_2^{en}(a,\Delta)+q_1(a-\Delta)$. So:

$$\begin{aligned}q_2^{ye}(a,\Delta) &= \frac{\pi(a)V_1(a-\Delta+1)-\pi(a-\Delta)V_1(a+1)}{\pi(a-\Delta)-\pi(a)}+V_2(a+1,\Delta)\\ &= \frac{\pi(a)q_1(a-\Delta)-\pi(a-\Delta)q_1(a)}{\pi(a-\Delta)-\pi(a)}+q_2^{en}(a,\Delta)+q_1(a-\Delta)\\ &= \frac{\pi(a-\Delta)}{\pi(a-\Delta)-\pi(a)}\left(q_1(a-\Delta)-q_1(a)\right)+q_2^{en}(a,\Delta)\end{aligned}$$

The three thresholds $q_2^{en}(a,\Delta)$, $q_2^{yn}(a,\Delta)$, and $q_2^{ye}(a,\Delta)$ can be unambiguously ordered for all a and Δ. First, $q_2^{yn}(a,\Delta)>q_2^{en}(a,\Delta)$ because $V_1(a-\Delta+1)>V_1(a+1)$. Second, $q_2^{ye}(a,\Delta)>q_2^{en}(a,\Delta)$ because $\frac{\pi(a-\Delta)}{\pi(a-\Delta)-\pi(a)}\left(q_1(a-\Delta)-q_1(a)\right)>0$. Third, $q_2^{ye}(a,\Delta)>q_2^{yn}(a,\Delta)$ because $V_2(a+1,\Delta)=q_2^{en}(a,\Delta)+q_1(a-\Delta)=q_2^{yn}(a,\Delta)+q_1(a)$, implying that $q_2^{yn}(a,\Delta)-q_2^{en}(a,\Delta)=q_1(a-\Delta)-q_1(a)$. As a result, $q_2^{ye}(a,\Delta)=\frac{\pi(a-\Delta)}{\pi(a-\Delta)-\pi(a)}\left(q_1(a-\Delta)-q_1(a)\right)+q_2^{en}(a,\Delta)>q_2^{yn}(a,\Delta)$.

Because $q_2^{en}(a,\Delta)<q_2^{yn}(a,\Delta)<q_2^{ye}(a,\Delta)$ for all a and Δ, the decision rule involving $q_2^{yn}(a,\Delta)$

never binds. Therefore, the optimal policy is:[2]

- Offer y if $q \geq q_2^y(a,\Delta)$.
- Offer e if $q \in \left[q_2^e(a,\Delta), q_2^y(a,\Delta)\right)$.
- Offer neither if $q < q_2^e(a,\Delta)$.

where $q_2^e(a,\Delta) \equiv q_2^{en}(a,\Delta)$ and $q_2^y(a,\Delta) \equiv q_2^{ye}(a,\Delta)$.

As in the one-daughter case, we solve for $q_2^e(a,\Delta)$ using backward induction. At $a = \bar{a}$, $V_2(a+1,\Delta) = V_1(a-\Delta+1)$ because $\pi(a+1) = 0$. Thus, $q_2^e(\bar{a},\Delta) = 0$. For $a \in [\underline{a}+\Delta, \bar{a})$:

$$\begin{aligned}
\mathbb{E}v_2^A(q,a,\Delta) &= \int_{\underline{Q}}^{q_2^e(a,\Delta)} \{q_2^e(a,\Delta) + q_1(a-\Delta)\}\, dF(\tilde{q}) \\
&+ \int_{q_2^e(a,\Delta)}^{q_2^y(a,\Delta)} \{\pi(a)\,(\tilde{q} + q_1(a-\Delta)) + (1-\pi(a))\,(q_2^e(a,\Delta) + q_1(a-\Delta))\}\, dF(\tilde{q}) \\
&+ \int_{q_2^y(a,\Delta)}^{\overline{Q}} \{\pi(a-\Delta)\,(\tilde{q} + q_1(a)) + (1-\pi(a-\Delta))\,(q_2^e(a,\Delta) + q_1(a-\Delta))\}\, dF(\tilde{q})
\end{aligned}$$

and:

$$\begin{aligned}
q_2^e(a,\Delta) &= V_2(a+1,\Delta) - V_1(a-\Delta+1) \\
&= \lambda \mathbb{E}v_2^A(q,a+1,\Delta) + (1-\lambda)\, v_2^N(a+1,\Delta) - V_1(a-\Delta+1) \\
&= \lambda \mathbb{E}v_2^A(q,a+1,\Delta) + (1-\lambda)\, V_2(a+2,\Delta) - V_1(a-\Delta+1) \\
&= \lambda \mathbb{E}v_2^A(q,a+1,\Delta) + (1-\lambda)\, (q_2^e(a+1,\Delta) + q_1(a-\Delta+1)) - q_1(a-\Delta)
\end{aligned}$$

which yields the following recursive solution for $a \in [\underline{a}+\Delta, \bar{a})$:

$$\begin{aligned}
q_2^e(a,\Delta) - q_2^e(a+1,\Delta) &= \lambda\pi(a+1)\psi\,(q_2^e(a+1,\Delta)) \\
&\quad + \lambda\,(\pi(a-\Delta+1) - \pi(a+1))\,\psi\left(q_2^y(a+1,\Delta)\right) \\
&\quad - \lambda\pi(a-\Delta+1)\psi\,(q_1(a-\Delta+1)) \\
q_2^y(a,\Delta) &= \frac{\pi(a-\Delta)}{\pi(a-\Delta) - \pi(a)}\,(q_1(a-\Delta) - q_1(a)) + q_2^e(a,\Delta)
\end{aligned}$$

with $q_2^e(\bar{a},\Delta) = 0$, as derived above. During $a \in [\underline{a}, \underline{a}+\Delta)$, the period before the younger sis-

[2] The placement of weak and strong inequalities is arbitrary because they represent indifference points.

ter reaches marriageable age, the elder sister's reservation quality evolves according to the one-daughter transition function: $q_2^e(a,\Delta) - q_2^e(a+1,\Delta) = \lambda\pi(a+1)\psi\left(q_2^e(a+1,\Delta)\right)$. Also, after one daughter's marriage, her sister's problem resets to the one-daughter problem above.

We now compare the reservation values of the elder and younger sisters to the reservation values they would have in the absence of a sister. First, we show that $q_2^y(a,\Delta) > q_1(a-\Delta)$. Note that $q_2^y(\bar{a},\Delta) = \frac{\pi(\bar{a}-\Delta)}{\pi(\bar{a}-\Delta)-\pi(\bar{a})} q_1(\bar{a}-\Delta) > q_1(\bar{a}-\Delta)$. Rearrange the two-daughter solution as follows:

$$\begin{aligned}
q_2^y(a,\Delta) - q_2^y(a+1,\Delta) &= \lambda\pi(a-\Delta+1)\psi\left(q_2^y(a+1,\Delta)\right) \\
&\quad + \underbrace{\lambda\pi(a+1)\left(\psi\left(q_2^e(a+1,\Delta)\right) - \psi\left(q_2^y(a+1,\Delta)\right)\right)}_{>0} \\
&\quad + \underbrace{\frac{\pi(a)}{\pi(a-\Delta)-\pi(a)}\left(q_1(a-\Delta) - q_1(a-\Delta+1)\right)}_{>0} \\
&\quad + \underbrace{\frac{\pi(a-\Delta)}{\pi(a-\Delta)-\pi(a)}\left(q_1(a-\Delta+1) - q_1(a)\right)}_{\geq 0} \\
&> \lambda\pi(a-\Delta+1)\psi\left(q_2^y(a+1,\Delta)\right)
\end{aligned}$$

For a given reservation quality and age, the increase in reservation quality from reducing age by 1 is larger for a younger daughter than for an only daughter. Therefore, the reservation quality-age profile for a younger daughter (while her elder sister is unmarried) is everywhere above that for an only daughter.

Second, we show that $q_2^e(a,\Delta) < q_1(a)$. Note from above that if $\pi(a-\Delta+1)\psi\left(q_1(a-\Delta+1)\right) > \left(\pi(a-\Delta+1) - \pi(a+1)\right)\psi\left(q_2^y(a+1,\Delta)\right)$, then $q_2^e(a,\Delta) - q_2^e(a+1,\Delta) < \lambda\pi(a+1)\psi\left(q_2^e(a+1,\Delta)\right)$. Because $q_2^y(a+1,\Delta) > q_1(a-\Delta+1)$, $\psi\left(q_1(a-\Delta+1)\right) > \psi\left(q_2^y(a+1,\Delta)\right)$, so the statement is true. For a given reservation quality and age, the increase in reservation quality from reducing age by 1 is smaller for a elder daughter than for an only daughter. During $a \in [\underline{a}, \underline{a}+\Delta)$, the transition function reverts to the one-daughter case, but $q_2^e(\underline{a}+\Delta,\Delta) < q_1(\underline{a}+\Delta)$, so the reservation quality for the elder daughter is everywhere lower than that for an only daughter. In summary, if q is log concavely distributed, then $q_2^e(a,\Delta) < q_1(a) < q_1(a-\Delta) < q_2^y(a,\Delta)$.

Two of the three claims in the Result hold in this model with no marriage-by-birth-order rule: (b) and (c). First, cumulative marriage risk is lower for a younger daughter than for an only daughter: $P_y(a, \Delta) = \sum_{\tilde{a}=\underline{a}}^{a} \lambda \pi(\tilde{a}) \left(1 - F\left[q_2^y(\tilde{a}, \Delta)\right]\right) \left(1 - P_y(\tilde{a}-1, \Delta)\right) < P_o(a)$. Second, expected spousal quality is lower for a girl with a sister than for an only daughter. This result follows from the fact that $\hat{q}_o = V_1(\underline{a})$, as shown in part (c) of the Proof of Result. Since expected spousal quality is equal to the continuation value at age $\underline{a}$, a deviation from the (unique) optimal policy must result in a reduction in expected quality.

Extension: The Sons Problem with Endogenous Search Intensity

The models above apply to daughters' marriages in rural South Asia. Because grooms' families typically exert search effort than brides' families, a model of sons' marriages in South Asia must allow for endogenous search intensity. To this end, suppose a family can influence the arrival rate at age a by exerting effort $e(a) \geq 0$, measured in units of disutility. The arrival rate is now $\lambda\left(e(a)\right)$, with $\lambda' > 0$, $\lambda'' < 0$, $\lambda\left(0\right)$ and $\lim_{e \to \infty} \lambda\left(e\right) < 1$. Let $V_1(a)$ be the expected value of the son at age a, before the realization of an offer. Let $v_1^F(q, a)$ be the expected value after finding a bride of quality q, but before she accepts. And let $v_1^N(a)$ be the expected value in the event that no bride is found. Then:

$$
\begin{aligned}
V_1(a) &= \lambda\left(e(a+1)\right) \mathbb{E} v_1^F(q, a) + \left(1 - \lambda\left(e(a+1)\right)\right) v_1^N(a) - e(a) \\
v_1^F(q, a) &= \max\left\{\pi(a)q + (1 - \pi(a))V_1(a+1), V_1(a+1)\right\} \\
v_1^N(a) &= V_1(a+1)
\end{aligned}
$$

The family chooses an optimal effort level $e_1(a)$ and a reservation quality $q_1(a)$. We first solve for $q_1(a)$ using backward induction. At $a = \bar{a}$, $V_1(a+1) = 0$ because $\pi(a+1) = 0$. Thus, $q_1(\bar{a}) = 0$. For $a \in [\underline{a} + \Delta, \bar{a})$:

$$
\mathbb{E} v_1^F(a) = \int_{\underline{Q}}^{q_1(a)} q_1(a) dF(\tilde{q}) + \int_{q_1(a)}^{\overline{Q}} \left\{\pi(a)\tilde{q} + \left(1 - \pi(a)\right) q_1(a)\right\} dF(\tilde{q})
$$

and:

$$
q_1(a) = V_1(a+1) = \lambda\left(e(a+1)\right) \mathbb{E} v_1^F(q, a+1) + \left(1 - \lambda\left(e(a+1)\right)\right) v_1^N(a+1) - e(a+1)
$$

which yields the recursive solution:

$$q_1(a) - q_1(a+1) = \lambda\left(e_1(a+1)\right)\pi(a+1)\psi\left(q_1(a+1)\right) - e_1(a+1)$$

with $q_1(\bar{a}) = 0$, as shown above. We now solve for $e_1(a)$ to maximize $V_1(a)$:[3]

$$\max_e \lambda\left(e\right)\mathbb{E}v_1^F(q,a) + \left(1-\lambda\left(e\right)\right)v_1^N(a) - e(a)$$

The first order condition is:

$$\lambda'\left(e_1(a)\right) = \frac{1}{\mathbb{E}v_1^F(q,a) - v_1^N(a)} = \frac{1}{\pi(a)\left(E[q|q > q_e(a,\Delta)] - q_e(a,\Delta)\right)} = \frac{1}{\pi(a)\psi\left(q_1(a)\right)}$$

Because $\lambda'' < 0$, the solution $e_1(a)$ is unique.[4]

Under a marriage-by-birth-order rule, $q_2(\bar{a},\Delta)$ solves $\mathbb{E}\left[q|q \geq q_2(\bar{a},\Delta)\right] = -q_1(\bar{a}-\Delta)$, so that $q_2(\bar{a},\Delta) < 0 = q_1(\bar{a})$. The transition equation is:

$$q_2(a,\Delta) - q_2(a+1,\Delta) = \lambda\left(e_2(a+1,\Delta)\right)\pi(a+1)\psi\left(q_2(a+1,\Delta)\right) - e_2(a+1,\Delta) - \left(q_1(a-\Delta) - q_1(a-\Delta+1)\right)$$

Optimal effort $e_2(a,\Delta)$ is determined by the same first order condition:

$$\lambda'\left(e_2(a,\Delta)\right) = \frac{1}{\mathbb{E}v_1^F(q,a) - v_1^N(a)} = \frac{1}{\pi(a)\psi\left(q_2(a,\Delta)\right)}$$

Because q is log-concavely distributed, the denominator is decreasing in $q_2(a,\Delta)$. So $e_2(a,\Delta) \gtreqless e_1(a) \iff q_2(a,\Delta) \lesseqgtr q_1(a)$. Suppose $e_2(a,\Delta) = e_1(a)$ and $q_2(a,\Delta) = q_1(a)$, which implies $V_2(a+1,\Delta) - V_1(a+1-\Delta) = V_1(a+1)$. But because the younger son is delayed by the elder son's presence in the household, this condition is impossible. Now suppose $e_2(a,\Delta) < e_1(a)$ and $q_2(a,\Delta) > q_1(a)$. But by raising $e_2(a,\Delta)$ and lowering $q_2(a,\Delta)$, the family could increase the value of the elder son's marriage search process (as shown in the one-son solution) while speeding the transition to the younger son's problem, thus contradictorily raising the family's expected lifetime

[3] The assumptions on $\lambda\left(\cdot\right)$ guarantee an interior solution.

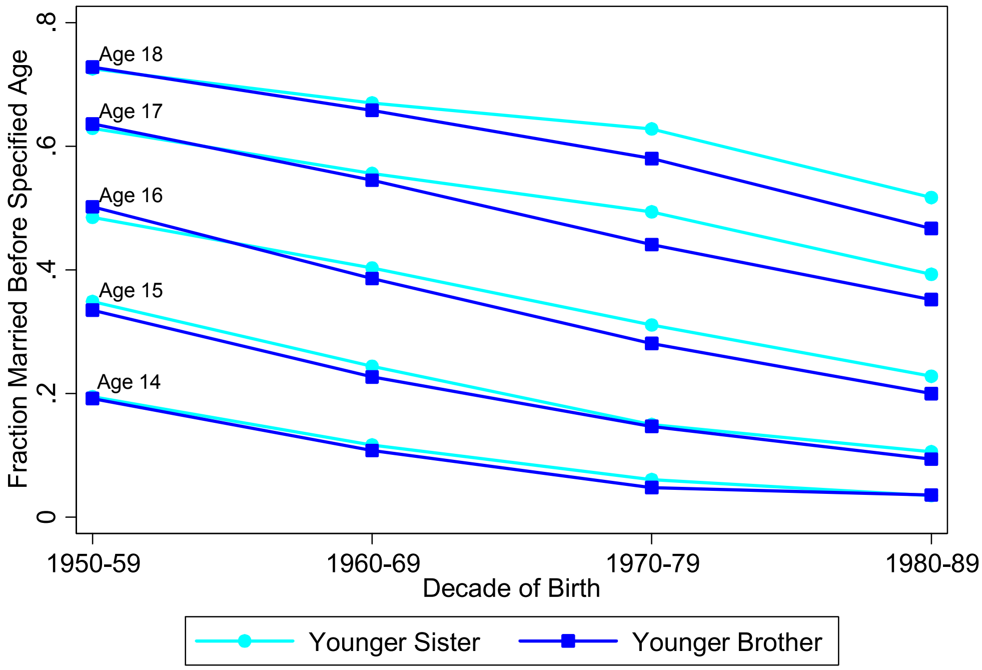
[4] The right-hand-side of the first order condition is declining in a if q is log-concavely distributed and $\delta(a) < 1 - \psi\left(q_1(a)\right)/\psi\left(q_1(a+1)\right)$. In this case, because $\lambda'' < 0$, $e_1(a)$ is increasing in a.

payoff. Therefore, $e_2(a, \Delta) > e_1(a)$ and $q_2(a, \Delta) < q_1(a)$.

The family channels some of the pressure from the younger sister into increased search effort. Due to effort adjustment, the elder brother's reduction in reservation quality is smaller in the model with endogenous search intensity than in the original model without. Nonetheless, the signs of the same-sex sibling effects are unchanged. If elder brothers must marry before younger brothers, then the presence of a younger brother increases a boy's cumulative marriage risk at any age and decreases expected his spousal quality, while the presence of an older brother decreases a boy's cumulative marriage risk at any age and decreases his expected spousal quality. The proof is similar to that for the daughters model.

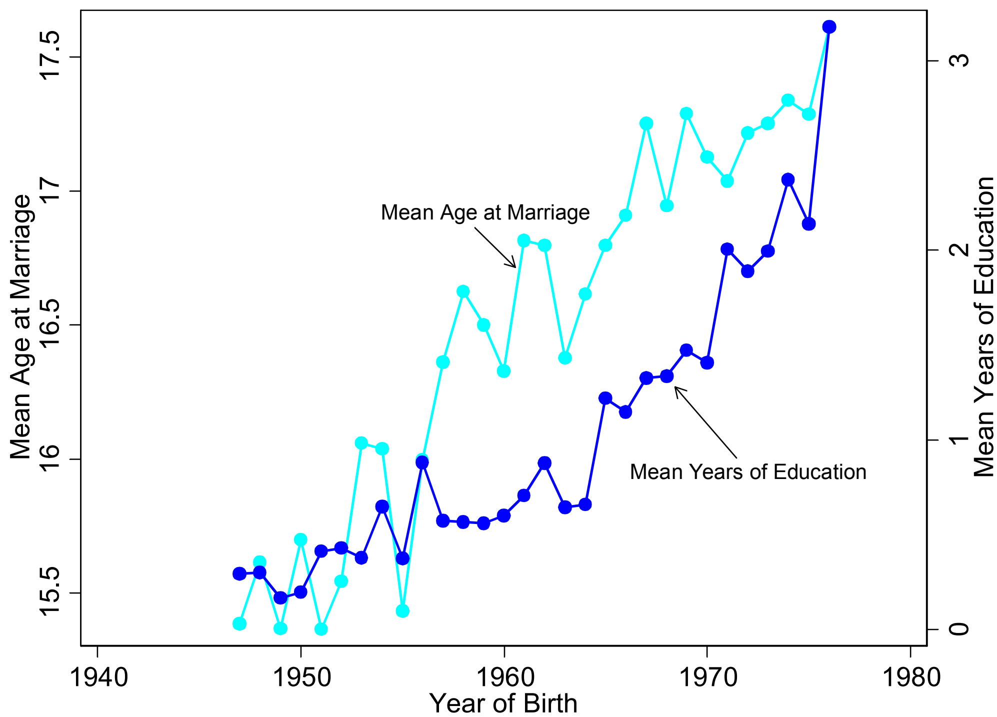
Marriage Institutions and Sibling Competition: Appendix Tables and Figures

Appendix Figure I
Early Marriage by Sex of Next-Youngest Sibling, Nepal



Notes: Sample includes all women aged 15-49 from the 2006 Nepal DHS and ever-married women aged 30-49 from the 1996 Nepal DHS. Women with no younger siblings are excluded, as are women born in the same year as a sibling and women with two next-youngest siblings born in the same year.

Appendix Figure II
Trends in Marriage Age and Educational Attainment, Women Aged 30-49, Nepal



Notes: Sample includes all women aged 30-49 from the 1996 and 2006 Nepal DHS.

Appendix Table I
Parental Coresidence and Marriage among Women and Men of Prime Marriageable Age, Post-1999 Pooled Data

	Women Aged 15-24				Men Aged 20-29		
A. Measure of Parental Coresidence: Mother Is a Survey Respondent							
	Mother Is a Survey Respondent	Mother Is Not a Survey Respondent			Mother Is a Survey Respondent	Mother Is Not a Survey Respondent	
Married	2,000 (*row=0.05, col=0.07*)	40,640 (*row=0.95, col=0.69*)	42,640 (*col=0.48*)	Married	6,345 (*row=0.20, col=0.29*)	25,321 (*row=0.80, col=0.52*)	31,666 (*col=0.45*)
Not Married	28,037 (*row=0.61, col=0.93*)	18,248 (*row=0.39, col=0.31*)	46,285 (*col=0.52*)	Not Married	15,196 (*row=0.39, col=0.71*)	23,510 (*row=0.61, col=0.48*)	38,706 (*col=0.55*)
	30,037 (*row=0.34*)	58,888 (*row=0.66*)			21,541 (*row=0.31*)	48,831 (*row=0.69*)	
B. Measure of Parental Coresidence: Linked to Mother Through Household Head							
	Linked	Not Linked			Linked	Not Linked	
Married	7,032 (*row=0.16, col=0.14*)	35,608 (*row=0.84, col=0.88*)	42,640 (*col=0.48*)	Married	18,153 (*row=0.57, col=0.36*)	13,513 (*row=0.42, col=0.67*)	31,666 (*col=0.45*)
Not married	41,628 (*row=0.90, col=0.86*)	4,657 (*row=0.10, col=0.12*)	46,285 (*col=0.52*)	Not married	31,987 (*row=0.82, col=0.64*)	6,719 (*row=0.17, col=0.33*)	38,706 (*col=0.55*)
	48,660 (*row=0.34*)	40,265 (*row=0.66*)			50,140 (*row=0.71*)	20,232 (*row=0.29*)	

Notes: The table reports frequencies with row and column fractions. Full linkage of all household members to coresident parents is not possible, so two proxies are used. In Panel A, the proxy is the participation of a household member's mother in an individual interview. The DHS conducts individual interviews with all female household members aged 15-49, so this proxy undercounts parental coresidence among individuals with mothers over 50. In Panel B, the proxy is whether a household member can be linked to a coresident mother through the household head. This proxy counts mothers of different ages equally but misses mother-child pairs who are not directly related to the household head. Source: DHS Fertility Histories. Pre-1999 surveys are excluded because they do not allow linkage between the household roster (which contains data on marriage) and the individual women's questionnaire (which contains the respondent's fertility history). All countries have at least one post-1999 survey.

Appendix Table II
Unadjusted Estimates of Next-Youngest Sister Effects

A. 15-24 Year-Old Women, All Countries

	Parental Cores.
Younger Sister	-0.032
	[0.003]
Number of Obs.	105,992

B. 15-19 Year-Old Women, Nepal 2006

	Marriage and Fertility (OLS Coefficients)		Own Human Capital (OLS Coefficients)	
	N. Married	Childless	In School	Illiterate
Younger Sister	-0.047	-0.020	-0.059	0.006
	[0.023]	[0.015]	[0.023]	[0.020]
Number of Obs.	1,945	1,945	1,944	1,945

C. 15-49 Year-Old Women, Nepal 2006

	Marriage and Fertility (Cox Hazard Ratios)		Own Human Capital (OLS Coefficients)		Health Outcomes (OLS Coefficients)	
	Marriage	1st Birth	Education	Illiterate	Height (cm)	BMI (m/kg^2)
Younger Sister	1.105	1.110	-0.133	0.026	0.092	-0.121
	[0.024]	[0.026]	[0.087]	[0.012]	[0.121]	[0.069]
Number of Obs.	8,706	8,706	8,706	8,701	8,669	8,667

D. 30-49 Year-Old Women, Nepal 1996 and 2006

	Marriage and Fertility (Cox Hazard Ratios)		Own Human Capital (OLS Coefficients)		Match Attributes (OLS Coefficients)			
	Marriage	1st Birth	Education	Illiterate	H's Ed.	H Skilled	H's Age	HH Assets
Younger Sister	1.041	1.042	-0.151	0.031	-0.234	-0.043	-0.177	-0.043
	[0.021]	[0.023]	[0.066]	[0.011]	[0.101]	[0.012]	[0.198]	[0.024]
Number of Obs.	6,882	6,882	6,882	6,878	6,677	6,807	6,334	6,882

Notes: Bivariate regressions. Brackets contain standard errors clustered by PSU. Samples include surviving women with at least one ever-born younger sibling. Only observations with singleton current and next births are included. Panel A is analogous to Tables I and III, Panel B to Table VIII, Panel C to Table IX, and Panel D to Table IX. Source: DHS Fertility Histories.

Appendix Table III
Predictors of Next-Youngest Sisters, Women Aged 15-24

	Bangladesh	India	Nepal	Pakistan
	(1)	(2)	(3)	(4)
Next Birth Interval (Yrs)	-0.0043	-0.0029	0.0026	-0.0054
	[0.0022]	[0.0011]	[0.0041]	[0.0039]
# Older Siblings	0.0053	-0.0054	-0.0111	0.0007
	[0.0049]	[0.0021]	[0.0070]	[0.0063]
# Older Brothers	-0.0035	0.0091	0.0114	-0.0044
	[0.0063]	[0.0030]	[0.0095]	[0.0088]
Mom's Age	-0.0011	0.0008	-0.0003	0.0014
	[0.0015]	[0.0006]	[0.0019]	[0.0019]
Mom's Highest	0.0004	-0.0008	0.0002	0.0019
Grade	[0.0018]	[0.0006]	[0.0032]	[0.0024]
Dad's Highest	0.0002	-0.0011	-0.0014	-0.0005
Grade	[0.0012]	[0.0005]	[0.0017]	[0.0015]
Joint F-test *p*-value	0.51	<0.01	0.59	0.76
Number of Obs.	14,650	77,589	7,542	6,211

Notes: OLS estimates. Brackets contain standard errors clustered at the PSU-level. Samples include surviving women with at least one ever-born younger sibling. Only observations with singleton current and next births are included. All regressions include fixed effects for birth year, survey year, maternal region of residence, and rural residence. The joint F-test only corresponds to the coefficients reported in the table. (The F-test for the entire regression yields similar *p*-values.) Source: DHS Fertility Histories.

Appendix Table IV
Pre-Birth Characteristics by Sex of Next-Youngest Sibling, Women Aged 15-24

	Bangladesh			India			Nepal		Pakistan	
	Brother	Sister		Brother	Sister		Brother	Sister	Brother	Sister
	(1)	(2)		(3)	(4)		(5)	(6)	(7)	(8)
Next Birth Interval (Yrs)	3.22	3.16	*	2.79	2.76	*	2.79	2.81	2.48	2.43
	[1.95]	[1.82]		[1.68]	[1.62]		[1.47]	[1.52]	[1.67]	[1.57]
# Older Sisters	0.84	0.86		0.70	0.69		0.81	0.77	0.91	0.92
	[1.07]	[1.08]		[0.97]	[0.96]		[1.01]	[0.99]	[1.16]	[1.15]
# Older Brothers	0.85	0.85		0.64	0.67	*	0.75	0.74	0.95	0.94
	[1.07]	[1.06]		[0.92]	[0.93]		[0.97]	[1.01]	[1.14]	[1.11]
Mom’s Age	40.39	40.42		40.95	40.97		41.84	41.71	41.96	42.00
	[4.78]	[4.85]		[4.43]	[4.44]		[4.37]	[4.33]	[4.44]	[4.58]
Mom’s Highest Grade	2.17	2.17		2.76	2.66	*	0.63	0.62	1.48	1.47
	[3.22]	[3.18]		[4.03]	[3.96]		[2.03]	[1.99]	[3.26]	[3.25]
Dad's Highest Grade	4.15	4.16		5.61	5.46	*	3.35	3.30	4.68	4.58
	[4.60]	[4.50]		[4.97]	[4.95]		[3.98]	[3.94]	[5.08]	[5.06]

Notes: Each cell reports a mean with a standard deviation in brackets. * denotes a difference in means that is significant at the 5% level. Samples include surviving women with at least one ever-born younger sibling. Only observations with singleton current and next births are included. Source: DHS Fertility Histories. Sample sizes are as reported in Appendix Table III.

Appendix Table V
Predictors of Next-Oldest Sisters, Women Aged 15-24

	Bangladesh	India	Nepal	Pakistan
	(1)	(2)	(3)	(4)
Next Birth Interval (Yrs)	-0.024	-0.022	-0.015	-0.030
	[0.004]	[0.002]	[0.006]	[0.006]
# Siblings Born Before Next-Oldest	0.007	0.013	-0.014	0.013
	[0.006]	[0.003]	[0.008]	[0.007]
# Brothers Born Before Next-Oldest	-0.013	-0.015	-0.004	-0.019
	[0.007]	[0.004]	[0.011]	[0.011]
Mom's Age	0.003	0.0020	0.0017	0.0030
	[0.002]	[0.0007]	[0.0024]	[0.0030]
Mom's Highest	0.001	0.0021	0.002	-0.001
Grade	[0.002]	[0.0007]	[0.004]	[0.003]
Dad's Highest	-0.001	-0.0002	0.0017	0.0028
Grade	[0.001]	[0.0005]	[0.0020]	[0.0017]
Joint F-test *p*-value	<0.01	<0.01	0.02	<0.01
Number of Obs.	11,742	61,133	5,688	4,767

Notes: OLS estimates. Brackets contain standard errors clustered at the PSU-level. Samples include surviving women with at least one ever-born younger sibling. Only observations with singleton current and next births are included. All regressions include fixed effects for birth year, survey year, maternal region of residence, and rural residence. The joint F-test only corresponds to the coefficients reported in the table. (The F-test for the entire regression yields similar *p*-values.) Source: DHS Fertility Histories.

Appendix Table VI
Pre-Birth Characteristics by Sex of Next-Oldest Sibling, Women Aged 15-24

	Bangladesh			India			Nepal			Pakistan		
	Brother	Sister		Brother	Sister		Brother	Sister		Brother	Sister	
	(1)	(2)		(3)	(4)		(5)	(6)		(7)	(8)	
Next Birth Interval (Yrs)	3.00	2.82	*	2.63	2.48	*	2.66	2.59	*	2.28	2.11	*
	[1.57]	[1.41]		[1.39]	[1.26]		[1.27]	[1.23]		[1.30]	[1.13]	
# Sisters Born Before	0.70	0.76	*	0.56	0.62	*	0.68	0.64		0.75	0.82	*
Next-Oldest	[0.95]	[1.01]		[0.86]	[0.91]		[0.93]	[0.89]		[1.04]	[1.09]	
# Brothers Born Before	0.75	0.77		0.60	0.61	*	0.66	0.61	*	0.82	0.82	
Next-Oldest	[0.99]	[1.00]		[0.88]	[0.89]		[0.93]	[0.89]		[1.04]	[1.02]	
Mom’s Age	42.13	42.16		42.45	42.49		43.24	43.03	*	42.95	43.10	*
	[4.23]	[4.32]		[4.04]	[4.01]		[3.84]	[3.87]		[4.07]	[4.11]	
Mom’s Highest Grade	2.04	2.02		2.46	2.56	*	0.50	0.58		1.38	1.38	
	[3.09]	[3.07]		[3.77]	[3.83]		[1.78]	[1.92]		[3.06]	[3.14]	
Dad's Highest Grade	4.14	4.07		5.33	5.39	*	3.07	3.28	*	4.45	4.63	
	[4.61]	[4.45]		[4.86]	[4.92]		[3.83]	[3.98]		[4.92]	[5.10]	

Notes: Each cell reports a mean with a standard deviation in brackets. * denotes a difference in means that is significant at the 5% level. Samples include women with at least one ever-born older sibling. Only observations with singleton current and last births are included. Source: DHS Fertility Histories. Sample sizes are as reported in Appendix Table V.

Appendix Table VII
Regressions of Marriage and Human Capital Outcomes on the Sex of the Next-Oldest Sibling, Nepal

A. 15-19 Year-Old Women, Nepal 2006

	Marriage and Fertility (OLS Coefficients)		Own Human Capital (OLS Coefficients)	
	N. Married	Childless	In School	Illiterate
Older Sister	0.048	0.030	0.041	-0.052
	[0.024]	[0.017]	[0.026]	[0.020]
Number of Obs.	1,883	1,883	1,882	1,883

B. 15-49 Year-Old Women, Nepal 2006

	Marriage and Fertility (Cox Hazard Ratios)		Own Human Capital (OLS Coefficients)		Health Outcomes (OLS Coefficients)	
	Marriage	1^{st} Birth	Education	Illiterate	Height (cm)	BMI (m/kg^2)
Older Sister	0.944	0.933	0.228	-0.035	-0.010	-0.007
	[0.021]	[0.023]	[0.092]	[0.011]	[0.121]	[0.070]
Number of Obs.	8,195	8,195	8,195	8,188	8,155	8,153

C. 30-49 Year-Old Women, Nepal 1996 and 2006

	Marriage and Fertility (Cox Hazard Ratios)		Own Human Capital (OLS Coefficients)		Match Attributes (OLS Coefficients)			
	Marriage	1^{st} Birth	Education	Illiterate	H's Ed.	H Skilled	H's Age	HH Assets
Older Sister	0.964	0.965	0.122	-0.022	0.042	0.012	-0.025	0.025
	[0.022]	[0.023]	[0.068]	[0.012]	[0.116]	[0.013]	[0.157]	[0.027]
Number of Obs.	6,197	6,197	6,197	6,191	6,013	6,101	5,714	6,197

Notes: Brackets contain standard errors clustered by PSU. Samples include surviving women with at least one ever-born older sibling. Only observations with singleton current and last births are included. All regressions control for religion, spacing from the respondent's birth, the year the respondent's mother initiated childbearing, and birth and survey year fixed effects. The OLS regressions include fixed effects for the exact composition of older siblings by birth order and sex; the Cox models stratify by this variable (so as not to impose proportionality). Source: 1996 and 2006 Nepal DHS Sibling Histories.

Appendix Table VIII
Next-Youngest Sister Effects on Female Outcomes in Childhood

	Bangladesh	India	Nepal	Pakistan
	(1)	(2)	(3)	(4)
A. Under-5 Mortality, All Births within 25 Years of Survey				
Younger Sister	0.007	0.004	0.002	-0.003
	[0.003]	[0.001]	[0.004]	[0.004]
Mean Among Girls w/ a Younger Brother	0.15	0.12	0.15	0.10
Number of Obs.	46,946	246,215	27,064	23,059
B. Height-for-Age Z-scores, Ages 2-4				
Younger Sister	-0.005	0.005	-0.082	0.157
	[0.065]	[0.040]	[0.064]	[0.157]
Mean Among Girls w/ a Younger Brother	-2.26	-1.82	-2.29	-2.50
Number of Obs.	1,993	5,429	1,594	657
C. Weight-for-Height Z-scores, Ages 2-4				
Younger Sister	-0.098	-0.024	0.015	0.121
	[0.041]	[0.028]	[0.042]	[0.089]
Mean Among Girls w/ a Younger Brother	-0.94	-0.95	-0.83	-0.40
Number of Obs.	1,993	5,429	1,594	657
D. School Attendance, Ages 5-9				
Younger Sister	0.014	0.002	-0.005	-0.007
	[0.009]	[0.006]	[0.013]	[0.016]
Mean Among Girls w/ a Younger Brother	0.81	0.50	0.70	0.58
Number of Obs.	5,364	16,512	4,171	3,373

Notes: OLS estimates. Brackets contain standard errors clustered at the PSU-level. Samples include girls with at least one ever-born younger sibling. Only observations with singleton current and next births are included. All regressions include fixed effects for age, mother's region of residence, survey year, and the exact composition of older siblings by birth order and sex. Regressions also control for spacing from the previous birth, maternal and paternal educational attainment, maternal age, religion, and rural residence. Source: DHS Fertility Histories.